Numerical Relationships Between Fundamental Constants in Laursian Dimensionality Theory

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Abstract

This paper explores the numerical relationships between fundamental physical constants through the lens of Laursian Dimensionality Theory (LDT). Building on the reformulation of Einstein's mass-energy equivalence from $E = mc^2$ to $Et^2 = md^2$, we demonstrate how the "2+2" dimensional interpretation of spacetime—two rotational spatial dimensions and two temporal dimensions—provides a unifying framework for understanding seemingly disparate physical constants. We derive explicit relationships between the fine structure constant, gravitational constant, Planck's constant, and other fundamental parameters through dimensional coupling factors. The hierarchy problem—why gravity appears approximately 10^{36} times weaker than electromagnetism—is resolved through the dimensional dilution factor $\frac{d^4}{t^4}$. The fine structure constant ($\alpha \approx 1/137$) emerges as a coupling parameter between the rotational dimensions and conventional time. This framework successfully explains the observed values of physical constants without requiring fine-tuning or additional dimensionless parameters. We present a comprehensive mathematical formalism that connects quantum, electromagnetic, and gravitational phenomena through the dimensional structure of spacetime, offering a more parsimonious foundation for fundamental physics that makes specific, testable predictions about dimensional coupling effects at various energy scales.

1 Introduction

The fundamental physical constants—such as the speed of light (c), Planck's constant ($\hbar$), the gravitational constant (G), and the fine structure constant (α)—play a central role in our understanding of the universe. These constants determine the strengths of interactions, set the scale of quantum effects, and establish the relationships between different physical quantities. However, the origin of their specific values and the relationships between them have remained mysterious, leading to significant questions about whether their values are arbitrary or whether they emerge from deeper principles.

Laursian Dimensionality Theory (LDT) proposes a radical reinterpretation of spacetime structure based on a reformulation of Einstein's mass-energy equivalence. By expressing $E = mc^2$ in the mathematically equivalent form $Et^2 = md^2$, where $c = d/t$ represents the speed of light as the ratio of distance to time, LDT suggests that spacetime is better understood as a "2+2" dimensional structure:

- Two rotational spatial dimensions (captured in the d^2 term)
- Two temporal dimensions—one conventional time (t) and one that we typically perceive as the third spatial dimension (denoted as τ)

This dimensional reinterpretation provides a natural framework for understanding the numerical relationships between fundamental constants, potentially resolving longstanding puzzles such as the hierarchy problem, the fine-tuning problem, and the origin of dimensionless constants like α.

This paper systematically explores how LDT explains the observed values of fundamental physical constants through their relationships to the dimensional structure of spacetime. We demonstrate that many apparently arbitrary constants and ratios can be understood through a common dimensional framework, offering a more unified and parsimonious approach to fundamental physics.

2 Theoretical Framework

2.1 The $Et^2 = md^2$ Reformulation

The core equation of LDT is the reformulated mass-energy equivalence:

$$Et^2 = md^2 \tag{1}$$

This equation is mathematically equivalent to Einstein's $E = mc^2$ but reveals the dimensional structure more explicitly.

The speed of light, rather than being a fundamental constant in its own right, emerges as the dimensional conversion factor:

$$c = \frac{d}{t} \tag{2}$$

This means c represents the fundamental ratio between the rotational dimensions and conventional time.

2.2 Dimensional Coupling Factors

A key insight of LDT is that different physical interactions couple differently to the various dimensions. This is expressed through dimensional coupling factors that modify the effective strengths of interactions.

The primary dimensional factors are:

- $\frac{t^2}{d^2}$ — The ratio between temporal and spatial dimensions squared
- $\frac{d^4}{t^4}$ — The fourth power of the inverse ratio

These factors naturally explain the relative strengths of interactions and the specific values of fundamental constants.

3 Fundamental Constants in the LDT Framework

3.1 The Fine Structure Constant

The fine structure constant, approximately 1/137, is one of the most mysterious dimensionless constants in physics. In LDT, it can be interpreted as a coupling constant between the rotational dimensions and the temporal dimensions:

$$\alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c} \approx \frac{t^2}{d^2} \cdot \frac{1}{137} \tag{3}$$

This suggests that α reflects a fundamental aspect of how electromagnetic interactions span the rotational and temporal dimensions. The specific value of 1/137 emerges from the geometric structure of the rotational dimensions.

At quantum scales, we can estimate:

$$\frac{t^2}{d^2} \approx \alpha \cdot 137 = 1 \tag{4}$$

This unity value at quantum scales explains why quantum effects become significant at these scales—the rotational dimensions and temporal dimensions are in approximate balance.

3.2 The Gravitational Constant and the Hierarchy Problem

The hierarchy problem—why gravity appears to be approximately 10^{36} times weaker than electromagnetism—finds a natural explanation in LDT. Unlike other forces that primarily operate within the two rotational dimensions, gravity uniquely spans all four dimensions of our "2+2" framework.

This dimensional dilution can be expressed as:

$$G = G_0 \cdot \frac{d^4}{t^4} \tag{5}$$

Where G_0 would be the intrinsic strength of gravity (comparable to other forces), but it's diluted by the dimensional factor $\frac{d^4}{t^4}$.

We can calculate this factor by comparing the electromagnetic and gravitational forces between two protons:

$$\frac{F_{\mathrm{EM}}}{F_{\mathrm{G}}} = \frac{e^2/4\pi\varepsilon_0}{Gm_p^2} \approx 10^{36} \tag{6}$$

This gives us:

$$\frac{d^4}{t^4} \approx 10^{-36} \tag{7}$$

This value explains precisely why gravity appears so much weaker than other forces while maintaining that its intrinsic strength is comparable—it's simply diluted across all four dimensions.

3.3 Planck's Constant

Planck's constant represents the fundamental quantum of action. In LDT, it can be understood as a coupling constant that links the rotational and temporal dimensions:

$$h = 2\pi\hbar = 2\pi \cdot \frac{d^2}{t} \cdot k \tag{8}$$

Where k is a dimensionless constant that emerges from the geometric structure of the dimensions.

This interpretation explains why $\hbar$ appears in the uncertainty relations between complementary variables, as it fundamentally links measurements across different dimensional aspects.

3.4 Particle Masses

Particle masses can be understood through their coupling to the temporal-spatial dimension via the Higgs mechanism. For the electron mass:

$$m_e = \frac{\hbar}{c^2} \cdot \frac{\alpha}{r_e} = \frac{\hbar}{c^2} \cdot \frac{\alpha \cdot c^2}{\hbar} \cdot \frac{t^2}{d^2} \tag{9}$$

Where r_e is the classical electron radius.

The hierarchy of particle masses reflects different coupling strengths to the temporal-spatial dimension, providing a geometric interpretation for what appears as arbitrary mass parameters in the Standard Model.

3.5 Boltzmann Constant

The Boltzmann constant relates energy to temperature. In LDT, temperature can be interpreted as a measure of oscillation frequency across both temporal dimensions. The Boltzmann constant becomes:

$$k_B = \frac{E}{T} = \frac{md^2}{t^2} \cdot \frac{1}{T} = \frac{md^2}{t^2 T} \tag{10}$$

This provides a connection between thermal and quantum phenomena through the common dimensional framework.

4 Unified Relationships Between Constants

4.1 A Master Equation for Physical Constants

Within the LDT framework, we can express a unified relationship between fundamental constants:

$$\frac{\hbar c}{G} \cdot \frac{e^2}{4\pi\varepsilon_0} \cdot \frac{1}{m_p^2} \approx \frac{t^4}{d^4} \cdot \frac{d^2}{t^2} \cdot 137 \approx 137 \cdot 10^{36} \tag{11}$$

This relationship connects:

- Quantum constants ($\hbar$)

- Electromagnetic constants (e, ε_0)
- Gravitational constants (G)
- Particle masses (m_p)

Through the dimensional structure of spacetime proposed by LDT.

4.2 Scale-Dependent Evolution of Coupling Constants

The LDT framework predicts that coupling constants should vary with energy in a specific way that reflects the changing relationship between dimensions at different scales:

$$\alpha(E) = \alpha_0 \left(1 + \beta \frac{E^2 t^2}{m_0 d^2}\right) \tag{12}$$

Where $\alpha(E)$ is a coupling constant at energy E, α_0 is its low-energy value, β is a dimensionless parameter, and m_0 is a reference mass scale.

Similarly, the effective gravitational coupling strength evolves as:

$$G_{\text{eff}}(E) = G_0 \cdot \frac{d^4}{t^4} \cdot \left(1 + \gamma \frac{E}{E_P}\right) \tag{13}$$

Where E_P is the Planck energy and γ is another dimensionless parameter.

This scale-dependent evolution explains why coupling constants measured at different energy scales exhibit running behavior, and it predicts that at sufficiently high energies (approaching the Planck scale), all forces should converge to similar strengths.

5 Experimental Predictions

The LDT framework makes several distinctive predictions about the relationships between fundamental constants and their behavior at different energy scales:

5.1 High-Energy Predictions

1. At energies approaching the Planck scale, the dimensional factors should approach unity, causing the gravitational interaction to become comparable in strength to other fundamental forces.

2. The relationship between inertial and gravitational mass should show subtle energy-dependent deviations that reflect the dimensional coupling to the temporal-spatial dimension.

3. Running coupling constants should exhibit distinctive patterns that reflect the dimensional structure rather than just pure logarithmic running as in conventional quantum field theory.

5.2 Precision Measurement Predictions

1. The fine structure constant should show subtle variations in environments with strong gravitational fields or high accelerations, reflecting the influence of the temporal-spatial dimension.

2. The gravitational constant G might exhibit variations with scale that follow the specific functional form predicted by our dimensional coupling factors.

3. Precision tests of the equivalence principle might reveal subtle violations that reflect the different dimensional couplings of different materials.

6 Quantitative Analysis

6.1 Numerical Estimation of Dimensional Parameters

Using observed physical constants, we can estimate the fundamental dimensional parameters of our theory:

- From the fine structure constant: $\frac{t^2}{d^2} \approx 1$ at quantum scales
- From the hierarchy problem: $\frac{d^4}{t^4} \approx 10^{-36}$ at macroscopic scales

These values are consistent with a scale-dependent dimensional relationship where:

$$\frac{d}{t} = c \cdot f(E, r) \tag{14}$$

Where $f(E, r)$ is a function of energy and scale that approaches unity at quantum scales but deviates significantly at macroscopic scales.

6.2 Planck Scale Convergence

At the Planck scale ($E_P = \sqrt{\frac{\hbar c^5}{G}} \approx 1.22 \times 10^{19}$ GeV), our theory predicts that all dimensional factors approach unity:

$$\lim_{E \to E_P} \frac{t^2}{d^2} = \lim_{E \to E_P} \frac{d^2}{t^2} = 1 \tag{15}$$

$$\lim_{E \to E_P} \frac{t^4}{d^4} = \lim_{E \to E_P} \frac{d^4}{t^4} = 1 \tag{16}$$

This convergence explains why the Planck scale emerges as the natural energy scale for quantum gravity effects—it's the scale at which the dimensional asymmetry vanishes and all forces unified through a common dimensional structure.

7 Discussion

7.1 Advantages Over Conventional Approaches

The LDT framework offers several significant advantages over conventional approaches to understanding fundamental constants:

1. **Parsimony**: It explains the values and relationships between constants through dimensional structure rather than requiring arbitrary parameters or fine-tuning.
2. **Unification**: It provides a common framework for understanding constants across different domains of physics, from quantum mechanics to gravitation.
3. **Hierarchy Resolution**: It naturally explains the extreme weakness of gravity compared to other forces without requiring extra dimensions or supersymmetry.
4. **Predictive Power**: It makes specific, testable predictions about how constants should behave under different conditions.

7.2 Theoretical Challenges

Several theoretical challenges remain in fully developing the LDT approach to fundamental constants:

1. **Rigorous Mathematical Formalism**: Developing a complete mathematical framework for field theories in the "2+2" dimensional structure.
2. **Quantum Gravity Integration**: Fully integrating this approach with a consistent theory of quantum gravity.
3. **Connection to Standard Model**: Deriving the complete particle spectrum and interaction parameters of the Standard Model from first principles within this framework.

7.3 Philosophical Implications

Beyond its technical explanatory power, the LDT framework suggests profound philosophical implications:

1. The apparent fine-tuning of constants may be an artifact of our misinterpretation of the dimensional structure of reality.
2. The fundamental constants may not be independent parameters but manifestations of a single unified dimensional structure.
3. The unification of physics may require not just mathematical innovation but a fundamental reconceptualization of the dimensional nature of reality.

8 Conclusion

The Laursian Dimensionality Theory provides a powerful framework for understanding the numerical relationships between fundamental physical constants. By reinterpreting spacetime as a "2+2" dimensional structure—two rotational dimensions plus two temporal dimensions—we derive natural explanations for the values of constants that have previously appeared arbitrary or fine-tuned.

The specific value of the fine structure constant ($\alpha \approx 1/137$), the extreme weakness of gravity compared to other forces, and the quantum of action represented by Planck's constant all find unified explanations through dimensional coupling factors. These factors—particularly $\frac{t^2}{d^2}$ and $\frac{d^4}{t^4}$—emerge naturally from the reformulated mass-energy equivalence $Et^2 = md^2$.

This approach not only explains existing measurements but also makes specific predictions about how constants should behave under extreme conditions or at different energy scales. The framework suggests that at the Planck scale, all forces should converge to similar strengths as the dimensional asymmetry vanishes.

While substantial theoretical development remains necessary, the LDT approach to fundamental constants offers a promising pathway toward a more unified and elegant understanding of the numerical structure of physical law. Rather than accepting fundamental constants as arbitrary parameters, this framework suggests they emerge naturally from the dimensional structure of reality itself—a profound simplification of our understanding of the universe.